Prepayment Risk and Equity Release Mortgages

The long-term and fixed nature of the cashflows produced by equity release mortgages is an important feature of the asset class for insurance firms who use these assets to back long-term fixed cashflow liabilities such as annuities. However, like most other forms of mortgage, an equity release mortgage usually provides the borrower with the option to prepay the mortgage prior to its intended maturity (which, in this case, is when the borrower dies or enters long-term care).

Many forms of bond and loan provide the borrower with some form of prepayment option. This is not a feature that is peculiar to the equity release mortgage asset class. It is sometimes the case that the prepayment option of a fixed rate bond or loan will include a 'make whole' provision or 'Spens' clause which adjusts the amount payable at prepayment to account for how changes in market interest rates have impacted on the present value of the schedule of outstanding loan repayments. This clause is intended to ensure that the contractual cashflow(s) of the loan can be re-constructed by re-investing the prepayment proceeds at the then-prevailing market interest rate. This mechanism can help ensure that prepayment risk does not create duration or cashflow mis-match risk for investors who are using the loan as part of a matching strategy for some fixed liability cashflows.

Some equity release mortgage products' prepayment terms include such a make whole feature. But there has been a trend in in recent years towards a simpler equity release mortgage product design that specifies fixed prepayment amounts that do not vary with the level of interest rates at the time of prepayment. Whilst this is simpler for the borrower to understand, it provides the borrower with a potentially valuable option, and creates what an actuary would call an anti-selection risk: if interest rates fall, the borrower may have an incentive to prepay the loan and thereby escape the fixed rate that was set at a time of higher interest rates. And, in the absence of an interest rate-dependent prepayment adjustment, this option has an economic cost for the lender who will have to reinvest the proceeds at a lower available yield.

Conceptually, this prepayment option has value to the equity release mortgage borrower, and this is a value that must arise at the expense of the lender. In 'real-life', there are many complexities that make the implications of this optionality difficult to analyse and measure, such as:

- The costs and risks associated with this optionality crucially depend on the degree to which the borrower is assumed to behave rationally and / or does not incur other costs when exercising their prepayment option.
- The mortgage may give the borrower the right to prepay at more than one prepayment date, and perhaps even at any time.
- Insurance firms usually securitise their ERM portfolios. In such cases, it will be necessary to determine how the costs and risks of prepayment of the underlying mortgages are distributed across the capital structure of the securitisation.
- The nature of the prepayment risk of an equity release mortgage is not determined only by interest rate risk. House price changes also impact on the present value of the mortgage's contractual cashflow, and hence impact on the financial consequences of prepayment.

This note will not consider the complexities that arise from securitisations or borrower (ir)rationality. The following analysis will consider how interest rate risk and house price risk impact on the cost of the prepayment option in the presence of rational borrower behaviour, and how this feeds into equity release mortgage valuation and its sensitivities to interest rates and house prices.

Prepayment and House Price Risk

We begin our analysis by first considering the impact that house price risk has on the cost of the prepayment option. So, for now, we will assume interest rates are deterministic (and we will return to the impact of interest rate uncertainty further below).

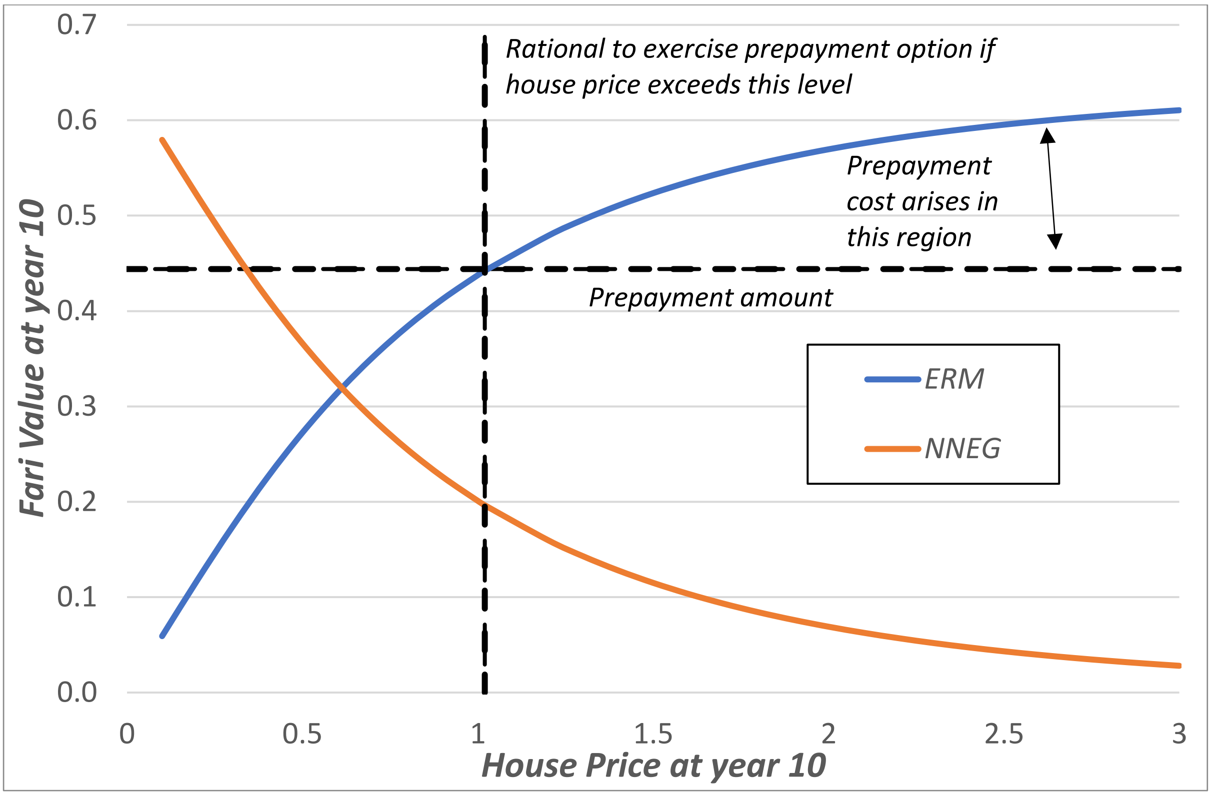
In general**, the borrower will have an incentive to prepay the mortgage when the prepayment amount is less than the present value of the outstanding loan payments**. House price risk is relevant to equity release mortgage prepayment risk if the mortgage has a No-Negative Equity Guarantee (NNEG). The presence of the NNEG means that the value of the maturity loan payment at any given point in time is a function of the level of the house price at that time. If the house price rises (falls), the value of the mortgage, in the absence of prepayment, increases (reduces). This means the lender will have a greater incentive to prepay when their house price has increased significantly since the origination of the mortgage (and vice versa). In such circumstances the borrower is paying a mortgage rate for a loan LTV that is higher than the actual current LTV of their loan. It therefore makes economic sense to prepay the mortgage and re-finance at a lower mortgage rate (or perhaps re-arrange their wider finances so that re-financing the mortgage is not necessary in order to prepay). Of course, this is bad news for the lender. It implies that the good-quality loans will tend to prepay, leaving their mortgage book with the riskier loans that have developed higher LTVs (for example, due to poor idiosyncratic house price performance).

We now develop a stylised example to develop insight into the impact of this form of prepayment risk. To do this, a very simple equity release mortgage will be used that will be familiar to those who have read my previous equity release mortgage articles. We will assume this mortgage does not have mortality or long-term care decrements and has a fixed and known maturity date of 25 years from origination (unless it is prepaid). We will assume the risk-free rate is 1.5% at all terms, and that the house that the mortgage is written on has a deferment rate of 3.5%. The starting Loan-to-Value ratio of the mortgage is 30% and the mortgage rate is 4.0%. For simplicity, this note will not consider the effects of illiquidity and will therefore assume all illiquidity premia are zero. If our simple example mortgage does not include a prepayment option, then the above assumptions imply that a house price volatility of 19% is required to obtain a fair value at origination equal to the loan amount.

Now let's begin our prepayment analysis gently by supposing the mortgage offers the borrower the right to prepay the mortgage at a single date, 10 years from origination. The prepayment amount is pre-determined as the loan balance that applies at that point in time. Assuming a starting house price of 1, a loan amount of 0.3, and a mortgage rate of 4.0%, the loan balance at the 10-year prepayment date is a fixed amount, $0.3 \times (1.04)^{10} = 0.444$.

So, the 10-year prepayment amount is set at 0.444. If the fair value of the ERM is greater than 0.444 at the prepayment date, the borrower ought to prepay. If the fair value is less than this amount, the borrower ought to keep the mortgage. If interest rates are deterministic, the value of the mortgage at the prepayment date will depend only on the house price at the prepayment date (assuming all our other assumptions are unchanged). The chart below shows how the value at year 10 of the equity release mortgage, and its NNEG, behave as a function of the house price at year 10. Note these are the values that would apply if the borrower did *not* prepay.

Exhibit 1: ERM and NNEG Values at Year 10 (assuming no prepayment)



The ERM at year 10 can be considered as the pre-NNEG value less the NNEG value at time 10. As the risk-free interest rate is assumed (so far) to be deterministic, the pre-NNEG value at time 10 is always $0.3 \times (1.04)^{25} / (1.015)^{15} = 0.640$. The borrower should prepay whenever the fair value of the mortgage at year 10 is greater than 0.444. This occurs whenever the year 10 NNEG value is less than 0.640 – 0.444 = 0.196. The chart shows that this occurs when the year 10 house price is greater than 1.02 (represented by the vertical dashed line).

Put another way, the mortgage lender is short the NNEG and the borrower is long the NNEG; the borrower's right to prepay can be considered as the option to sell the NNEG back to the lender at a specified price. In this example, the right to prepay is equivalent to a 10-year put option on the NNEG (which itself is a put option on the house) with a strike price of 0.196.

So, in our simple example, the prepayment option can be considered as a well-defined option on an option. Analytical solutions for the valuation of such 'compound' options are available. It is therefore possible that there is a neat analytical solution to the total ERM value in the presence of this single prepayment option. However, the following analysis is going to take the (arguably!) easier route of valuing the mortgage using stochastic simulation modelling. This approach is more flexible and will allow us to more easily explore complications such as multiple prepayment dates and stochastic interest rates.

The table below summarises the valuation results produced by the stochastic simulation model (using 10,000 simulations with antithetic variables).

Exhibit 2: Valuation results with and without the 10-year prepayment option

	Without prepayment option	With 10-yr prepayment option
'Pre-NNEG' Maturity Value	0.550	0.413
NNEG Value	-0.250	-0.223
Prepayment Cashflow Value	-	0.095
ERM Value	0.300	0.285
ERM House Price Delta	+0.18	+0.14
ERM Effective Duration	10 years	7 years

Let's unpack these numbers. The left-hand column shows the results when the mortgage has no prepayment option. As noted above, the pre-NNEG cashflow in the absence of prepayment is 0.550 and the model has been calibrated to produce a NNEG value of 0.250 such that the ERM value at origination is 0.300, the loan amount. The results of the stochastic simulation model are reassuringly consistent with the Black-Scholes calculations that we used to calibrate the valuation basis (i.e. to set the house price volatility to 19%).

The simulation model can be used to estimate the sensitivities of the mortgage value to changes in the house price and change in the level of the risk-free interest rate[1]. The mortgage's house price delta measures how much the mortgage value changes for a small change in the house price. We would expect the mortgage's house price delta to be positive. That is, an increase in the house price will increase the value of the mortgage. We could put it another way – an increase in the house price reduces the value of the NNEG, and as the mortgage is short the NNEG, this increases the value of the mortgage. The model generates an estimate for the delta of the (no prepayment option) mortgage with respect to the house price of +0.18. That is, a £1 increase in the house price will increase the mortgage value by 18p. Note this estimate of the delta is only directly applicable for small changes in the house price. The delta itself changes with the house price; the relationship between the mortgage value and the house price is non-linear.

The sensitivity to interest rates is measured by the effective duration of the mortgage. Technically, we have defined this as $\frac{-1}{V}\frac{dV}{dr}$. The simulation model can again readily provide an estimate of this measure of interest rate sensitivity. In the absence of the NNEG, the effective duration of this 25-year mortgage is, naturally enough, 25 years. You might be surprised to see that the effective duration of our example 25-year mortgage is only 10 years. **The interest rate sensitivity of the NNEG value reduces the effective duration of the mortgage very materially**.

Why? In the presence of the NNEG, an increase (decrease) in the interest rate not only results in a higher (lower) discount rate being applied to the mortgage cashflow, it also, for any given size of house risk premium, increases (decreases) the expected size of the mortgage cashflow (as derivative valuation assumes house price inflation is equal to the risk-free interest rate plus an arbitrary house risk premium less the deferment rate). These discounting and cashflow effects move in opposite directions and, as a result, can result in a materially reduced duration for the mortgage. The size of the impact depends on the LTV of the mortgage. If the LTV is very low, the cashflow effect will be small and the duration of the mortgage will be similar to the duration of a risk-free loan. If the LTV is very high, the mortgage becomes very similar to deferred possession of the house, and its value

[1] A range of methods can be used to do estimate valuation sensitivities using stochastic simulation methods, here we have used the simple 'bump and revalue' approach.

does not depend on the risk-free rate but only on the deferment rate (and so has an interest rate duration of zero).

Now let's consider the effect of adding a prepayment option with a single prepayment date of 10 years from origination and a fixed prepayment amount equal to the 10-year loan balance. We assume that the prepayment option is exercised by the borrower when the fair value of the mortgage at year 10 (if not prepaid) exceeds the 10-year loan balance (i.e. the prepayment amount). As per Exhibit 1, we know this will occur whenever the house value at year 10 exceeds 1.02.

So, in this case the ERM will produce one of two cashflows: the prepayment cashflow of 0.444 at year 10, or the ERM maturity cashflow at year 25 (which will be the lower of the year 25 house price and the accrued loan balance of 0.800). Exhibit 2 above shows that the value of these cashflows sum to 0.285. **The total mortgage value has been reduced from 0.300 to 0.285 (5%) by the introduction of the 10-year prepayment option.**

Interestingly, the ERM value's sensitivities to house price changes and interest rate changes are materially affected by the presence of the prepayment option. **The mortgage's house price delta has been reduced from +0.18 to +0.14 and its effective duration has been reduced from 10 years to 7 years.**

Why has the presence of the prepayment option reduced the mortgage's house price delta? The prepayment option is valuable to the borrower and therefore costly to the lender in scenarios where the house price *rises*. This is opposite to the form of house price exposure created by the NNEG (i.e. the NNEG exposes the lender to house price *falls*). The prepayment option therefore provides some offset to the house price risk created by the NNEG and this results in an overall reduction in the mortgage value's sensitivity to changes in the house price. Similar economic logic can also explain why the effective duration of the mortgage has been reduced: the presence of the prepayment option (without a make whole clause) means that some of the lender's economic participation in the value created by low rates is reduced, and this is 'priced in' and results in a reduced mortgage value sensitivity to changes in interest rates. But the duration effect can also be thought of in a straightforward way: prepayment reduces the expected term of the cashflows.

All of the above valuation analysis has been undertaken using risk-neutral simulation modelling. None of the above valuation results are sensitive to the property risk premium assumption (we have not assumed it is zero, we have simply used a method that takes advantage of the fact that the valuation is invariant to the size of the assumed risk premium). If we make an explicit property risk premium assumption, we can also make some statements about probabilities and expectations. For example, a property risk premium of 3.5% implies the average LTV at year 10 (before prepayment) will be 55%. We know from above that the year 10 prepayment option will be exercised whenever the LTV is less than 0.444 / 1.02 = 43.5% at years 10. This happens with a probability of 46%. As a result of this selection effect, **the prepayment option increases the expected LTV of the remaining mortgage at year 10 from 55% to 77%**.

Other Prepayment Dates

The above analysis has been re-run for a range of different *single* prepayment terms from 1 year to 20 years. In each case the prepayment amount is assumed to be the loan balance at that time and the prepayment is assumed to occur whenever the prepayment amount is less than the fair value of the loan at that time.

Exhibit 3 shows the results obtained for the various prepayment dates. When considering the pattern of the results, it is useful to note that the case of no prepayment option being available on the 25-year mortgage is equivalent to a 25-year prepayment term.

Exhibit 3: Valuation results with and without a single prepayment option of various terms

	With 1-yr prepay option	With 5-yr prepay option	With 10-yr prepay option	With 15-yr prepay option	With 20-yr prepay option	Without prepay option
'Pre-NNEG' Maturity Value	0.323	0.378	0.413	0.436	0.451	0.550
NNEG Value	-0.159	-0.200	-0.223	-0.238	-0.246	-0.250
Prepay Value	0.127	0.106	0.095	0.089	0.087	-
ERM Value	0.290	0.284	0.285	0.288	0.293	0.300
ERM House Price Delta	+0.14	+0.12	+0.14	+0.16	+0.16	+0.18
ERM Effective Duration	3 years	7 years	7 years	10 years	10 years	10 years

The above results suggest the mortgage value is not particularly sensitive to the term of the prepayment option: all prepayment terms between 1 and 20 years reduce the value of the mortgage by between 2.5% and 5.0%. The probability of the prepayment option being exercised is also reasonably invariant across the choice of prepayment term – in all cases the probability is between 46% and 49% (with the 3.5% property risk premium assumed above). But the valuation sensitivities are materially different, and they differ in ways that are broadly intuitive given the above discussion of the 10-year prepayment results.

Introducing Interest Rate Risk into the Prepayment Analysis

The above results suggest that the presence of a prepayment option can have a material impact on the value of the mortgage and the value's sensitivity to interest rates and the house price. However, thus far we have only considered the impact that *house price risk* has on the prepayment option. That is, so far we have assumed the risk-free interest rate is *fixed* at 1.5%.

Stochastic interest rates can be introduced into our valuation model in a relatively straightforward way. But there are a couple of points to note. First, as the mortgage has cashflows that may arise at more than one point in time, we need a stochastic model of the whole yield curve, rather than merely one point on the yield curve. There is a very large library of well-established stochastic yield curve models that are regularly used in derivative valuation theory and practice, so we just need to pick one. For the purposes of this exercise, we will use one of the simplest – the 1-factor Vasicek model.

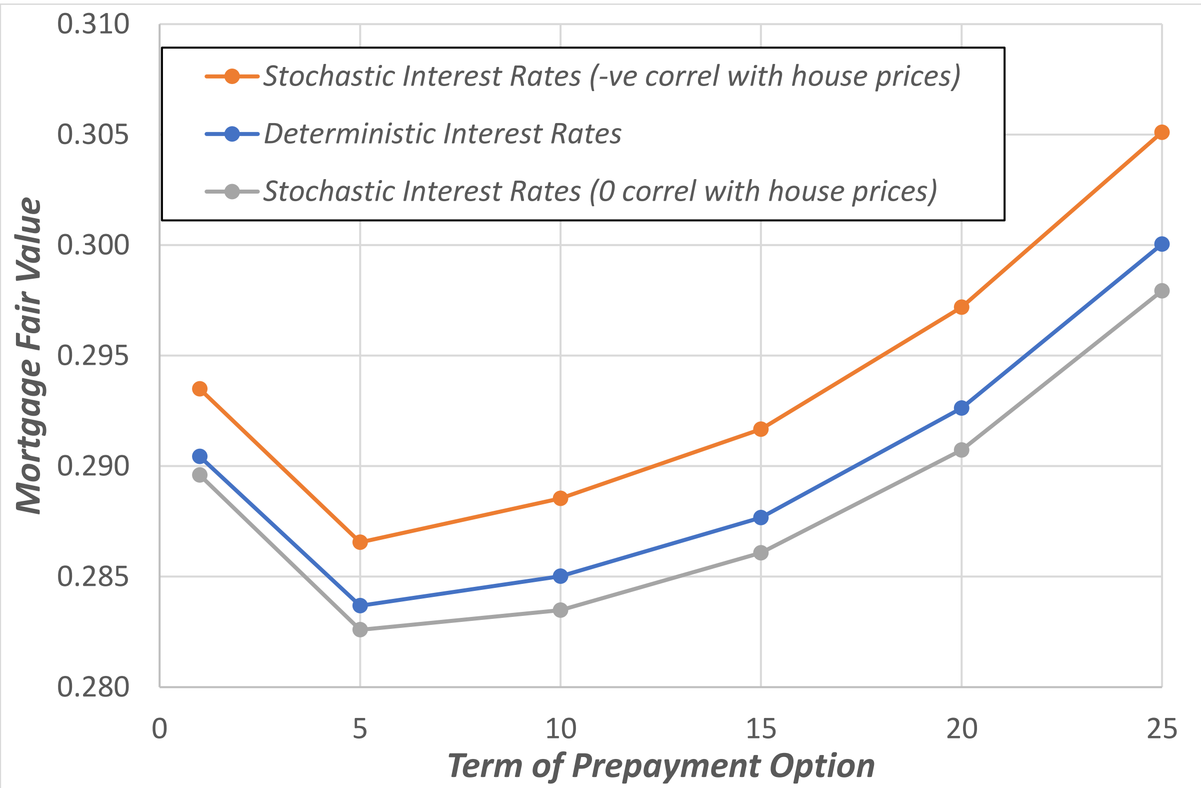
The second point to note is that the fair value of the mortgage at any prepayment date will now, of course, depend on the interest rate at that point in time. So we will retain the same prepayment decision rule – if the prepayment amount is less than the fair value of the loan, the borrower will prepay – but now the fair value will vary with both the house price and the relevant interest rate at the prepayment date.

We also need to assume a correlation between the stochastic shocks to the house price and the shocks to the yield curve. Here we assume a correlation of -0.3. This means that a downward shock in interest rates is more likely to coincide with a positive house price change than if the two shocks were independent.

Finally, we need a calibration of the stochastic yield curve model. Conventionally, when using the model to value something, we would select model parameters that fit well to the observable prices of assets that share some similarities with the valuation asset (so, in our case, interest rate options such as swaptions might be a natural candidate). For the purpose of our example, however, a simpler approach has been taken where we have used an illustrative calibration that could be regarded as providing a reasonable description of interest rate dynamics[2].

The chart below shows the valuation results that are obtained for the mortgage values with various (single) prepayment terms, with and without stochastic interest rates. In this chart, the no-prepayment case has been plotted as a 25-year prepayment term (which is the same thing when maturity is 25 years). Two sets of stochastic interest rate results have been produced – one that assumes interest rates and house prices are independent, and the other that assumes the correlation of -0.3 described above.

Exhibit 4: Valuation results with and without a single prepayment option of various terms



[2] Specifically, the 1-factor Vasicek parameters used in this example are r0 = 0.015; μ = 0.0155; σ = 0.01; α = 0.3. This model and calibration imply the short-term interest rate is mean-reverting and normally distributed with a 1-year volatility of 1%.

There are a few points to note in this chart. First, if interest rates and house prices are assumed to have uncorrelated shocks, then a stochastic yield curve reduces the value of the mortgage. This is, I think, intuitive – interest rate uncertainty is another source of risk that contributes to the overall risk that the NNEG is exposed to (as it creates some added variability in house price inflation); this increases the value of the NNEG and hence reduces the value of the mortgage. However, the size of this effect is quite small in our example. In the simplest case where there is no prepayment option (equivalent to the 25-year prepayment term in the chart), the presence of uncorrelated stochastic interest rates reduces the mortgage value from 0.300 to 0.298 (0.7%). Perhaps more importantly in the context of our current discussion, the presence of uncorrelated stochastic interest rates does not result in the prepayment option having a materially bigger impact on the mortgage value.

The other interesting feature of this chart is that negatively correlated interest rate and house price shocks actually result in a *higher* mortgage value (i.e. lower NNEG values). Why? Well, the scenarios with low interest rates will now have, on average, higher house prices than when rates and house prices are assumed to be uncorrelated. This translates into, on average, higher mortgage cashflows in those scenarios. The cashflows produced in these low interest rate scenarios are discounted at a lower discount function, and hence this effect results in a higher average present value.

So, this suggests the correlation between interest rates and house prices has a crucial impact on the nature of the impact of stochastic interest rates on the mortgage value. However, the magnitude of all of these stochastic interest rate effects is quite small in our example. It may be that the simple stochastic interest rate model and calibration that has been used in this example is understating the contribution that interest rate uncertainty may have on mortgage values and prepayment optionality. The calibration assumes a 1-year volatility for the short rate of 1%. The effects of the assumed level of mean-reversion in rates, however, means that the probability distribution of the 15-year spot rate in year 10 has a standard deviation of only 0.31%. An alternative calibration[3] of the Vasicek model was considered which generates substantially more interest rate volatility. In this case, the standard deviation of the 15-year spot rate in year 10 is 0.79% instead of 0.31%. But the ERM values with and without the 10-year prepayment option barely change relative to the results produced by our first Vasicek model calibration.

This lack of sensitivity to interest rate volatility is perhaps surprising, but it is driven by a number of factors: the NNEG of an ERM means that its interest rate sensitivity is materially lower than we might expect (e.g. 10 year effective duration for a 25-year mortgage); interest rate risk and property risk diversify to a significant degree; and, perhaps most importantly, the volatility of an individual residential property is high relative to the variability of a long-term interest rate.

Given the observed lack of materiality of the effect of stochastic interest rates, the remainder of this note will assume the risk-free interest rate is deterministic.

Before we continue with our prepayment analysis, we make one further reflection on the immateriality of the stochastic interest rate effect: as noted in the introductory paragraphs, some equity release mortgages have been written with prepayment terms that do include an interest rate 'make whole' clause. This analysis suggests the benefit of such a clause is quite limited: the predominant prepayment risk is not driven by the risk of falls in interest rates, but by the 'risk' of increases in house prices. It would be fairly impractical for equity release mortgages to include prepayment terms that vary with the performance of the underlying house price since origination. It would require some form of house valuation to be conducted at the prepayment date, and the

[3] In the alternative calibration, r0 = 0.015; μ = 0.0175; σ = 0.015; α = 0.2.

prepayment value to be adjusted in light of it, and this is unlikely to be easy to implement or communicate to the borrower. (Although if interest rates and house prices were strongly negatively correlated, the interest rate make whole provision could do the job of usually producing a higher prepayment amount when the house price is higher. But the correlation would need to be quite strong for this to be effective.)

Introducing Multiple Prepayment Dates

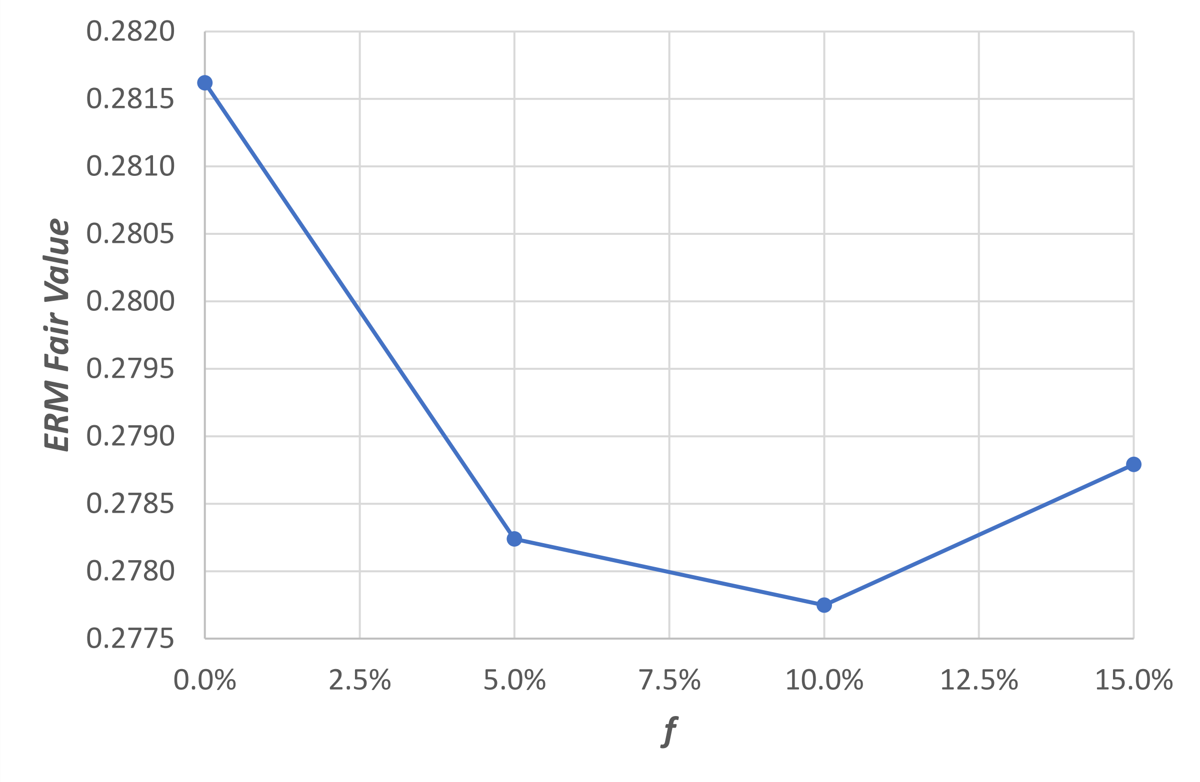
So far, we have only considered the case where the mortgage provides a *single* prepayment date (albeit a range of different ones). This section introduces the analysis of an equity release mortgage that provides the borrower with a choice of *multiple* prepayment dates. This complicates the analysis, mainly because the presence of multiple prepayment dates makes it harder to determine what the borrower's rational prepayment behaviour should be. When there is a single prepayment date, the rational prepayment decision is (theoretically) straightforward to identify: if the prepayment amount is less (greater) than the then fair value of the mortgage (when not prepaid), the borrower should (not) choose to prepay. But suppose we are considering this prepayment decision when the mortgage also provides one or more further prepayment dates in the future. In this case, the prepayment decision needs to make allowance for the fact that prepaying now gives up the option to prepay later. So, our rational prepayment decision rule now must be that we should prepay if the prepayment amount exceeds the fair value of the loan assuming no prepayment (as before) *plus* an allowance for the value of the option(s) to prepay in the future.

This suggests that the determination of the rational prepayment decision requires a form of backward recursive calculation. In our simulation modelling framework, this would translate into a nested stochastic calculation. For the purposes of this exploratory analysis, we will use a simpler and approximate approach to estimating the rational decision making rule in the presence of multiple prepayment dates: when at a prepayment date where future prepayment dates exist, we will assume that the threshold for deciding to prepay is higher than when no further prepayment options exist. Specifically, we will assume that the prepayment amount scaled by some constant factor greater than 1 (e.g. 1.05) must be less than the fair value assuming no prepayment in order for prepayment to occur.

We now consider the case where the 1-year, 5-year, 10-year, 15-year and 20-year prepayment dates all apply to the same mortgage. We model the prepayment decision with this constant factor approach. For example, at time 1 the prepayment amount is 0.3 x 1.04 = 0.312. So, if, at time 1, the fair value of the loan, assuming no prepayment, is greater than 0.312 x (1 + *f*), we assume the mortgage is prepaid. Otherwise, we carry on to year 5 and then consider again (where the prepayment amount will be 0.3×1.04^5) and so on for years 10 and 15. At year 20, we do not apply the factor uplift in the prepayment decision rule as there is no further prepayment date after year 20.

The chart below shows how today's fair value of the mortgage varies with the factor uplift parameter *f*.

*Exhibit 5: Today's Mortgage Fair Value as a function of parameter **f***



Let's first consider where $f = 0$. In this case, the borrower is assumed to prepay at the first prepayment date at which the fair value of the loan (assuming no prepayment) exceeds the prepayment amount. This results in a mortgage fair value of 0.2816. This is a bit smaller than the mortgage fair values shown in Exhibit 3 when the mortgage had a single prepayment date. This is intuitive – the greater amount of prepayment optionality makes the prepayment option more valuable and hence reduces the mortgage value further (but only by another 1% or 2%). Exhibit 5 shows that a positive value for f between 5% and 15% reduces the fair value further. Again, this is an intuitive result – by the above discussion, we think the rational (i.e. optimal) prepayment rule will have $f > 0$. Of course, the model's 'true' optimal decision rule may be much more complex than the simple linear scalar rule that has been applied here. For example, it would seem likely that f should be greater at early terms when numerous future prepayment dates remain. So, it may well be that these results understate the value of the mortgage's prepayment option.

For the remainder of this analysis, we will consider the effects of the prepayment option when the prepayment decision rule uses $f = 0.1$. In this case, the mortgage value and sensitivities are broadly consistent with the results produced for the single prepayment dates. The mortgage value, as shown above in Exhibit 5, is 0.2777, which means **the prepayment option has reduced the value of the mortgage by 7% or so relative to the no-prepayment case**. These assumptions produce a house price delta of +0.10 and an effective duration of 6 years for the mortgage.

The Effect of Early Prepayment Charges

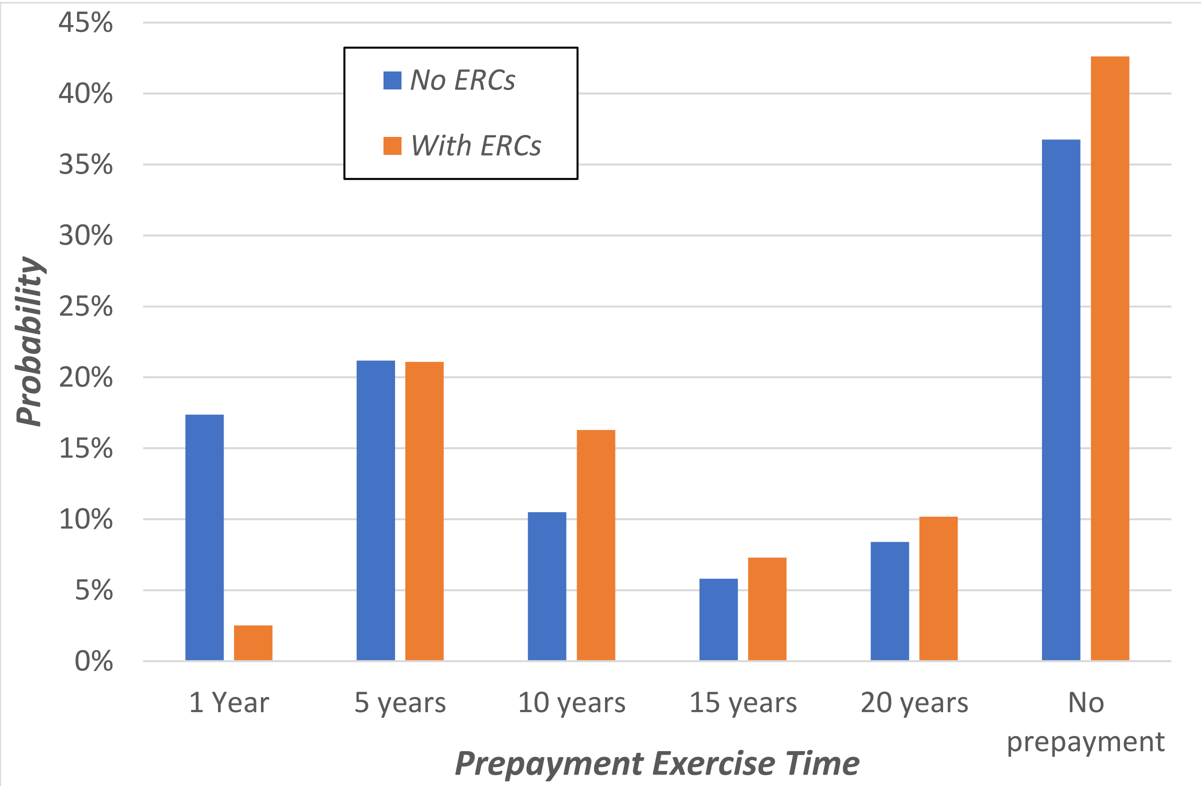
This section considers the effect that a schedule of early prepayment charges has on the above analysis. Virtually all mortgages include early prepayment charges. These are partly intended to compensate the mortgage lender for the costs incurred in acquiring the mortgage. But early prepayment charges also provide a natural means of charging the borrower for their prepayment option. The presence of an early prepayment charge has two effects on the above analysis:

- It increases the threshold for rational prepayment and therefore will tend to reduce prepayment frequency (all else being equal).
- It increases the size of the cashflow that the lender receives when the prepayment occurs. However, the prepayment event will always result in a value loss to the lender, as the borrower only prepays when the prepayment amount including the applicable early prepayment charge is less than the fair value of the loan.

We specify the early prepayment charge applicable at year *t* as a form of scaling factor that is applied to the year *t* prepayment amount. Specifically, we assume an early prepayment charge of 10% applies at the 1-year prepayment date; 5% at the 5-year prepayment date; and zero thereafter.

We can analyse the effect of these early prepayment charge on the probability distribution of the prepayment decision. As in the earlier discussion of probabilities, these probabilities depend on the assumed size of the property risk premium (though the mortgage value, house price delta and effective duration do not). Exhibit 6 below shows the probability distribution of the prepayment decision when the property risk premium of 3.5% is assumed.

Exhibit 6: Prepayment Probabilities; with and without Early Prepayment Charges



The effect of the early prepayment charges on prepayment timing is fairly intuitive. The large early prepayment charge that is applicable at year 1 significantly reduces the probability of exercise at year 1 from 17% to 3%. In the presence of the 10% year-1 early prepayment charge, a very large house price rise is required in the first year of the mortgage in order for prepayment at year 1 to be rational. In many scenarios though, the early prepayment charge merely defers the prepayment to a later point in time. As a result, the 10, 15 and 20-year prepayment probabilities are higher in the case with ERCs. Nonetheless, the probability of the mortgage being held to maturity is somewhat higher in the presence of ERCs (43% compared to 37% with no ERCs).

Exhibit 7 below shows the effect of early prepayment charges on the mortgage value and its sensitivities.

Exhibit 7: Valuation results with and without prepayment option and Early Prepayment Charges

	No prepayment	With prepayment, no ERCs	With prepayment and ERCs
ERM Value	0.300	0.278	0.281
ERM House Price Delta	+0.18	+0.10	+0.13
ERM Effective Duration	10 years	6 years	7 years

The table of results above suggests early repayment charges of the size typically charged in the ERM market only partially offset the cost of prepayment. They also partially offset the effect of prepayment on the mortgage fair value's sensitivities to house prices and interest rates.

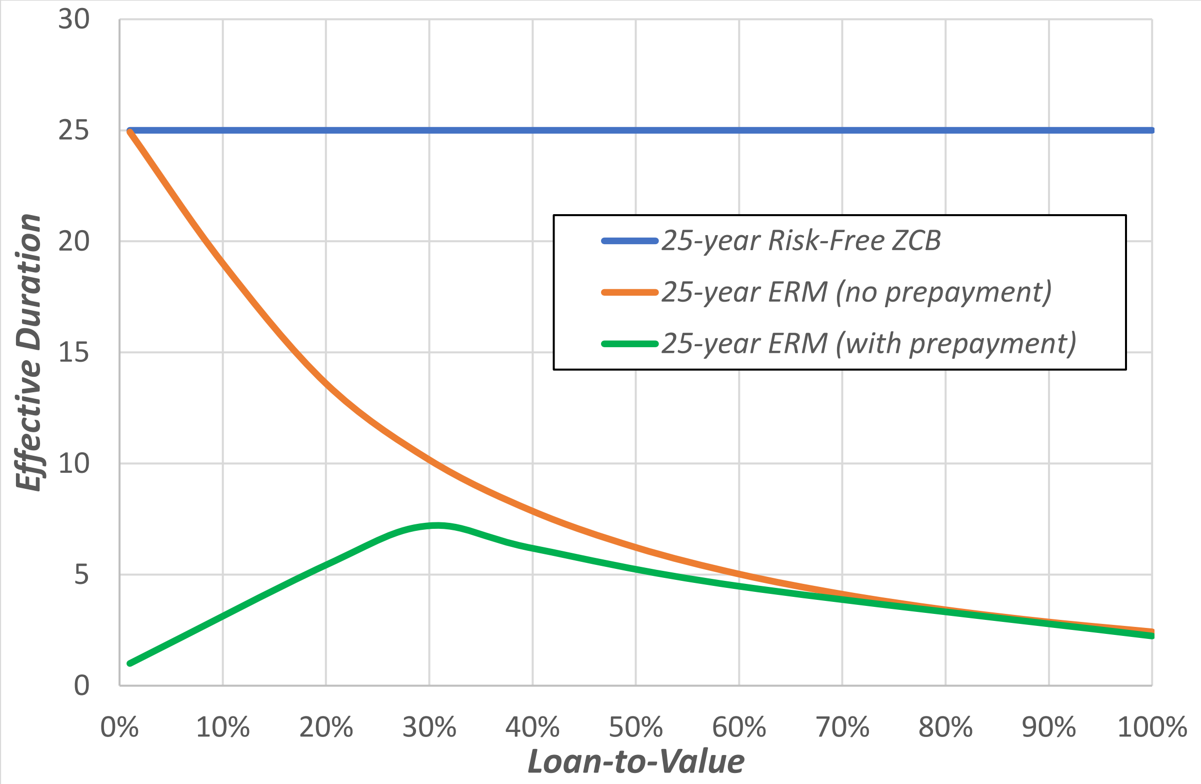
Further Thoughts on the Duration Behaviour of ERMs

The above illustrative analysis has suggested that the effective duration of a 25-year ERM is reduced from 10 years to 7 years by prepayment optionality in the presence of Early Prepayment Charges. It is important to note that this result has been obtained for a mortgage with a current LTV of 30%. The duration of the ERM, with and without the prepayment option, will vary very materially with LTV. For example, in the absence of a prepayment option, the mortgage's effective duration, which was shown to be 10 years at an LTV of 30%, will tend to 25 years as the LTV tends to zero (as the mortgage becomes a risk-free loan); and the effective duration will tend to zero as the LTV increases to very high levels (as the mortgage becomes deferred possession of the house).

When the mortgage includes the prepayment option, the effective duration will tend to that of the no-prepayment case as LTV increases, as the prepayment option will become less valuable in these circumstances. So, with prepayment, we also expect effective duration to tend to zero as LTV tends to very high levels. But in the case where the LTV of the mortgage falls to significantly lower levels, the presence of the prepayment optionality will fundamentally change the effective duration of the mortgage. Instead of the effective duration tending to the expected maturity of the mortgage (25 years in our example), it will instead fall to close to zero as imminent prepayment will become increasingly likely.

This behaviour can be illustrated by results obtained from our example model, as shown in Exhibit 8 below (the 'with prepayment' results have been produced using the same assumptions as the right-hand column of Exhibit 7, i.e. prepayment option applies at years 1, 5, 10, 15 and 20; Early Prepayment Charges of 10% at 1 year and 5% and 5 years apply; and an f parameter of 0.1 is used to determine the prepayment decision). The mortgage is originated with an LTV 0f 30% and the different LTV levels are produced by changing the current value of the house.

Exhibit 8: Effective Duration as a Function of LTV



The duration behaviour of ERMs may be problematic with or without a prepayment option. In the absence of prepayment, the economics of NNEG valuation means that our example 25-year mortgage has a starting effective duration of 10 years. If the loan is being used to cashflow match a 25-year liability cashflow, this may be a significant cause for concern. And this issue will be greater if the LTV increases significantly due to house price falls (as the effective duration will fall too). The presence of prepayment amplifies this difficulty. In our example it reduces the starting effective duration from 10 years to 7 years. Moreover, this difference in the effective duration of the with and without-prepayment cases will grow materially if the house price increases significantly, for the reasons explained above.

These duration characteristics could, for example, complicate the ability of an insurer to demonstrate that it can meet the PRA's Effective Value Test in a downward interest rate stress. Of course, in reality, it is the senior tranches of ERM securitisations that are used to match insurers' liability cashflows. This note has not discussed securitisation and how the costs and risks of prepayment are naturally propagated through the capital structure of a securitisation. However, economically, we would expect securitisation valuations to correspond with the equation of value. This means we can expect the capital structure of the securitisation, in aggregate, to exhibit the same interest rate sensitivity as that produced by the underlying mortgage portfolio. As a result, it seems unlikely that securitisation can fully immunise a matching strategy from the effects that NNEGs and prepayment options have on ERMs' effective duration.

Craig Turnbull FIA, 11th May 2020.